

DISCRIMINATIVE POWER OF A MODIFIED BONFERRONI'S TEST

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INTRODUCTION

The problem of choosing the more adequate statistical test for comparison of means of two treatments in research work is dependent on the type of error adopted, whether comparisonwise or experimentwise, the latter under general and/or partial null hypothesis.

It is well known that LSD's and Duncan's multiple range test are of the **comparisonwise type**, SNK's is of the **experimentwise type** under general null hypothesis, and that Tukey's, Bonferroni and Dunnett's tests are of the **experimentwise type** under general and/or partial null hypothesis (SAS, 1990).

Comparisonwise Type I error rate (H_0 true) is the ratio of the number of comparisons incorretly declared significant, divided by the total number of comparisons tested (STEEL & TORRIE, 1981).

Experimentwise Type I error rate (H_0 true) is the ratio of the number of experiments with one or more comparisons incorretly declared significant, divided by the total number of experiments conducted (STEEL & TORRIE, 1981).

The objective of the present research is to evaluate the discriminative power of a new test, called **Modified Bonferroni's Test** and compare it with the tests cited above.

From now on we are denoting Fisher's Test by (LSD), Duncan's by (D), Student Newman-Keuls by (SNK), Tukey's by (Tu), Bonferroni's by (B), Dunnett's by (Du) and the Modified

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Bonferroni's by (BM). Scheffé's test used more effectively for complex contrasts and Waller-Duncan's test, not very much used so far, were not included in this research. We also performed the LSD (protected t test) and Duncan's protected test (DP).

PERECIN & BARBOSA (1988) carried out a simulation on various tests and appointed conclusions that are corroborated for some of the tests here utilized.

MATERIAL AND METHODS

The design used was of the randomized block type.

The theoretical model used was:

$$Y_{ij} = M + T_i + B_j + E_{ij}; i = 1, 2, \dots, t; j = 1, 2, \dots, r.$$

The values of the parameters were: $M = 3000$, the T_i 's values were 40%, 30%, 20%, 10%, etc., of the M value.

Four groups, G_1 , G_2 , G_3 and G_4 were studied. For G_1 , $T_1 = 40\%$, $T_2 = 10\%$, $T_3 = 0\%$ and $T_4 = 0\%$ (control), where $k = 2$ ("a priori" selected comparisons with the control), $t=4$, $r=3$ and $r=6$ and $CV = 10\%$, $CV = 15\%$ and $CV = 20\%$ (coefficients of variation) of M . For G_2 , $T_1 = 30\%$, $T_2 = 20\%$, $T_3 = 0\%$ and $T_4 = 0\%$ (control), in all combinations, as above ($k=2$).

For G_3 , $T_1 = 40\%$, $T_2 = 30\%$, $T_3 = 20\%$, $T_4 = 10\%$, $T_5 = 0\%$ and $T_6 = 0\%$ (control). Now $k = 4$, $t = 6$, and all combinations were performed.

For G_4 , $T_1 = 40\%$, $T_2 = 30\%$, $T_3 = 20\%$, $T_4 = 10\%$, $T_5 = 0\%$, $T_6 = 0\%$ (control), $T_7 = 5\%$, $T_8 = 3\%$, $T_9 = 1\%$, $T_{10} = -5\%$, $T_{11} = -3\%$ and $T_{12} = -1\%$. Here, again, $k=4$, $t=12$, and all combinations of CV 's and r 's were performed.

The B_j 's values were the same in all sets.

The E'_{ij} s were obtained by the RANNOR FUNCTIONS of the SAS, 1990.

We performed 200 experiments for each combination when $r=3$ and 100 experiments for each combination when $r=6$, taking together 100 pairs of the single experiments.

On the whole 2,400 experiments for $r=3$ were carried out and 1,200 experiments for $r=6$ type. We used SAS Program (SAS, 1990) to perform the simulations, the analyses of variance and the tests already specified.

The comparisons of treatments 1 (40%), 2 (30%), 3 (20%) and 4 (10%) with the control 6 (0%) evaluates the discriminative power of the respective contrast; the comparison of treatment 5 (0%) with the control 6 (0%) evaluates the size of type I error of the test between treatments 5 and 6. Since treatments 5 and 6 are equal, their difference is a simultaneous measure of comparisonwise and experimentwise type I error.

THE MODIFIED BONFERRONI'S TEST

The usual Bonferroni's Test is used to establish confidence intervals or carry out a statistical test of significance for k comparisons chosen "a priori".

The means of two treatments, \bar{y}_1 and \bar{y}_2 , are considered different according to Bonferroni's test, if

$$|\bar{y}_1 - \bar{y}_2| \geq t(\gamma_B, df) \cdot s \sqrt{2/r} \quad (\text{SAS, 1990}).$$

The t is the Student t value for γ_B level of significance, df degrees of freedom, s the standard deviation and r the number of replications for each mean.

To calculate γ_B , if α is the joint level of significance for the k comparisons (with values 0.05 or 0.01), then, for each of the k comparisons, $\gamma_B = \alpha/k$.

If the number of treatments in the experiment is t , the γ_B value adequate for comparisons of all possible differences between two means is $\gamma_B = \alpha/t(t-1) \div 2$, where $k = C_t^2$ is the number of all possible combinations of t means taken two at a time.

In the modified Bonferroni's Test, for the comparisons of k differences between two means, we should start testing the general null hypothesis in the Analysis of Variance. If the null hypothesis is rejected at α level, that is, if F_o obtained is greater than the F_c critical level, we perform the Modified Bonferroni's Test.

The test assumes that $\gamma_{BM} = \alpha (1 + P(F))/k$ (A).

The calculation of $P(F)$ is as follows:

Let us assume that $F_o = MS$ treatments/ MS residual in the analysis of variance of the experiment.

We know that:

$$EMS \text{ treatments} = \sigma^2 + (r \sum t_i^2)/(t-1),$$

$$EMS \text{ residual} = \sigma^2.$$

If H_o (general null hypothesis) is true, then

$$T_1 = T_2 = \dots = T_t = 0.$$

If H_a (general alternative hypothesis) is true, at least one of the T_i 's is different from zero.

The non-centrality parameter $\lambda = r \sum T_i^2 / \sigma^2$ is a parameter of the non-central F distribution (WINNER et al., 1991).

In the analysis of the experiment,

$$F_o = (s^2 + r \sum T_i^2 / (t-1))/s^2 = 1 + r \sum T_i^2 / s^2 (t-1).$$

$$\text{An estimate } \hat{\lambda} \text{ of } \lambda \text{ is } \hat{\lambda} = r \sum T_i^2 / s^2 = (F_o - 1) (t-1).$$

The critical value of F for the rejection of H_o is the value F_c . If $F_o > F_c$ then H_o is rejected.

The probability of values smaller than F_o if H_a is true is:

$$P(F_o): \int_0^{F_o} g(F_o, t-1, (t-1)(r-1), \lambda) dx.$$

The probability of values smaller than F_c if H_a is true is:

$$P(F_c) = \int_0^{F_c} g(F_c, t-1, (t-1)(r-1), \lambda) dx.$$

The values may be calculated from PROBF FUNCTION of the SAS Program (SAS, 1990).

We define

$$\begin{aligned} P(F) &= P(F_o) - P(F_c) \\ &= \int_{F_c}^{F_o} g(F_c, t-1, (t-1)(r-1), \lambda) dx. \end{aligned}$$

which shows that $0 < P(F) < 1$.

As an example, assume that in an experiment, $r=3$, $df=10$, $t=6$, $F_o=9.72$, $F_c = 3.33$ and $k=4$. Since $F_o > F_c$ the null hypothesis shall be rejected. The calculation needed is:

$$\hat{\lambda} = (t-1)(F_o-1) = 5(9.72 - 1) = 43.60 ,$$

$$P(F_o) = \int_0^{F_o} g(9.72, 5, 10, 43.60) dx = 0.4682 ,$$

$$P(F_c) = \int_0^{F_c} g(3.33, 5, 10, 43.60) dx = 0.0155. \text{ Then:}$$

$$P(F) = P(F_o) - P(F_c) = 0.4527 .$$

Using formula (A) ,

$$\gamma_{BM} = 0.05 (1 + 0.4527)/4 = 0.01816 .$$

For the Bonferroni's test we have:

$$\gamma_B = 0.05/4 = 0.0125 .$$

The interval of variation of γ_{BM} is: $\gamma_B < \gamma_{BM} < 2\gamma_B$.

The argument to use γ_{BM} is based on the following reasoning: "If the F ratio MS treat/MS residual is large ($F_o > F_c$), the null hypothesis is rejected and there is "evidence" of the existence of heterogenous treatments; therefore we do not need to be so rigorous in using $\alpha = 0.05$ for the joint comparisons because we already accepted that H_o is false. We consider $P(F)$ as the weight of "evidence" that H_a is true. So we use $\alpha(1+P(F))$ as the joint level for k "a priori" comparisons.

$$\text{Then } \gamma_{BM} = \alpha (1 + P(F))/k .$$

This type of reasoning was already used in the Waller-Duncan's test (CHEW, 1977).

To be consistent we should use the Modified Bonferroni's Test (BM) only if H_o is rejected (H_o if false). The structure of the test proposed incorporates the "weight of evidence" of the veracity of H_a through the introduction of the F_o value obtained in the analysis of variance for the calculation of the t value, used to judge k chosen "a priori" comparisons of means. The results presented in Tables 1 to 6 regarding BM were obtained in this way.

In performing their test, Waller-Duncan calculate the test even if H_o is not rejected. It would be possible in the Modified Test to use $P(F)$ in the same way. As $P(F) = P(F_o) - P(F_c)$, if $F_o < F_c$ (H_o not rejected), $P(F)$ will be negative, and in this case $\gamma_{BM} < \gamma_B$; the test would be then more restrictive than Bonferroni's. When $P(F) = 0$, the two tests are identical.

RESULTS

We define as **Discriminative Power** of a test as the percentage of significant contrasts among k taken "a priori" contrasts of two means. In the Tables the cells represent the significant percentage of the comparison between the specified mean and the control mean obtained in a large group of experiments. The Tables 1 to 6 present the **discriminative power** of all tests performed, based in the conditions established in the research, the following conclusions are possible.

Table 1. Results obtained when $\alpha=3$, $CV = 10\%$, for $k=2$ and $k=4$, for groups $G_1 + G_2$, G_3 and G_4 , and for $df=6$ ($t=4$), $df=10$ ($t=6$) and $df=22$ ($t=12$), for differences of 40%, 30%, 20%, 10%, and 0% between the respective treatment mean and the control mean. The values in the cells represent the **discriminative power** in percentage based on 200 simulated experiments.

df	$T_1 - T_6$ (40%)			$T_2 - T_6$ (30%)			$T_3 - T_6$ (20%)		
	6	10	22	6	10	22	6	10	22
LSD	98	99	99	85	94	92	51	56	60.5
LSDP	92.5	97.5	98.5	75	93	92	49	56	60
D	98	99	97.5	83.5	91.5	88	50.5	51	49.5
DP	92.5	97.5	97.5	74.5	90.5	88	49	51	49.5
SNK	89.5	93.5	92	66.5	73	63	35	30.5	26
Tu	85.5	91	87.5	64	62.5	52	22.5	17	16.5
B	76.5	80.5	87	51	50	44.5	16	10.5	14
Du	91.5	95.5	94	69	71.5	73.5	30.5	26	33
BM	93	97.5	97	71	78	84.5	39	33	45

df	$T_4 - T_6$ (10%)			$T_5 - T_6$ (0%)		
	6	10	22	6	10	22
LSD	20.5	19	22.5	5.5	3.5	3.5
LSDP	20.5	19	22.5	5.5	3.5	3.5
D	20	18.5	16.5	5.25	3	2.5
DP	20	18.5	16.5	5.25	3	2.5
SNK	13	11.5	3.5	5	2	0
Tu	8	2	2.5	1	0	0
B	6.5	0.5	2	0.75	0	0
Du	12	8.5	6	1.75	1	0
BM	16	9	10	4	4	3

Table 2. Results obtained when $r=6$, $CV = 10\%$, for $k=2$ and $k=4$, for groups $G_1 + G_2$, G_3 and G_4 , and for $df=12$ ($t=4$), $df=20$ ($t=6$) and $df=44$ ($t=12$), for differences of 40%, 30%, 20%, 10%, and 0% between the respective treatment mean and the control mean. The values in the cells represent the **discriminative power** in percentage based on 100 experiments each one resultant of grouping two single experiments.

df	$T_1 - T_6$ (40%)			$T_2 - T_6$ (30%)			$T_3 - T_6$ (20%)		
	12	20	44	12	20	44	12	20	44
LSD	100	100	100	100	100	100	86	90	92
LSDP	100	100	100	100	100	100	86	90	92
D	100	100	100	100	99	99	85	87	87
DP	100	100	100	100	99	99	85	87	87
SNK	100	100	100	100	98	99	81	72	67
Tu	100	100	99	100	98	90	61	56	48
B	100	99	99	98	97	89	59	47	39
Du	100	100	100	100	98	99	71	72	66
BM	100	100	100	100	98	99	81	77	81

df	$T_4 - T_6$ (10%)			$T_5 - T_6$ (0%)		
	12	20	44	12	20	44
LSD	44	35	41	5.5	7	9
LSDP	44	35	41	5.5	7	9
D	41	35	36	5.5	7	1
DP	41	35	36	5.5	7	1
SNK	34	26	10	5.5	5	0
Tu	16	7	7	0	2	0
B	14	4	5	0	2	0
Du	28	12	20	1.5	2	0
BM	38	17	35	3.5	2	0

Table 3. Results obtained when $r=3$, $CV = 10\%$, for $k=2$ and $k=4$, for groups $G_1 + G_2$, G_3 and G_4 , and for $df=6$ ($t=4$), $df=10$ ($t=6$) and $df=22$ ($t=12$), for differences of 40%, 30%, 20%, 10%, and 0% between the respective treatment mean and the control mean. The values in the cells represent the **discriminative power** in percentage based in 200 simulated experiments.

df	$T_1 - T_6$ (40%)			$T_2 - T_6$ (30%)			$T_3 - T_6$ (20%)		
	6	10	22	6	10	22	6	10	22
LSD	77.5	86	90.5	58	58.5	66	29	31.5	39
LSDP	58.5	74	83.5	42.5	54	63	25	28	37.5
D	75	62	87	57	54	57.5	28	26.5	30.5
DP	57.5	53	81.5	42	50.5	55	24.5	24	29.5
SNK	53	54	55	34	28	21	16	11.5	6
Tu	47.5	48.5	43	29.5	20	14	14	6.5	4.5
B	36.5	27.5	32	21.5	16	10	8.5	4	2
Du	56	61	67.5	37.5	33.5	34	19	12.5	11
BM	56	62	78.5	41	34.5	45	17	13.5	18.5

df	$T_4 - T_6$ (10%)			$T_5 - T_6$ (0%)		
	6	10	22	6	10	22
LSD	12	6.5	14.5	5.75	4.5	6
LSDP	12	6	14.5	5.5	4	6
D	12	6	10	5.5	3	2
DP	12	5.5	10	5.5	2.5	2
SNK	8.5	2.5	1	4	2	0.5
Tu	5	0.5	0.5	1.25	0	0
B	3	0.5	0.5	1.25	0	0
Du	7.5	2	5	2	0	0
BM	9.5	3	7.5	2.5	1	2

Table 4. Results obtained when $r=6$, $CV = 15\%$, for $k=2$ and $k=4$, for groups $G_1 + G_2$, G_3 and G_4 , and for $df=12$ ($t=4$), $df=20$ ($t=6$) and $df=44$ ($t=12$), for differences of 40%, 30%, 20%, 10%, and 0% between the respective treatment mean and the control mean. The values in the cells represent the **discriminative power** in percentage based on 100 experiments from the grouping of two single experiments.

df	$T_1 - T_6$ (40%)			$T_2 - T_6$ (30%)			$T_3 - T_6$ (20%)		
	12	20	44	12	20	44	12	20	44
LSD	98	100	100	92	95	95	67	55	62
LSDP	93	100	100	89	95	95	67	55	62
D	98	100	99	91	93	91	64	49	53
DP	93	100	99	89	93	91	64	49	53
SNK	91	95	97	81	75	75	51	31	24
Tu	87	93	95	78	65	51	33	20	16
B	85	90	95	70	57	46	27	15	12
Du	93	96	98	80	73	76	45	30	29
BM	91	98	98	84	80	87	55	32	45

df	$T_4 - T_6$ (10%)			$T_5 - T_6$ (0%)		
	12	20	44	12	20	44
LSD	23	13	21	3.5	3	3
LSDP	23	13	21	3.5	3	3
D	20	11	20	3.5	3	3
DP	20	11	20	3.5	3	3
SNK	15	4	6	3.5	2	1
Tu	8	2	2	1.5	0	0
B	7	1	2	1	0	0
Du	12	4	9	3	1	1
BM	18	5	12	3.5	2	1

Table 5. Results obtained when $r=3$, $CV = 20\%$, for $k=2$ and $k=4$, for groups $G_1 + G_2$, G_3 and G_4 , and for $df=6$ ($t=4$), $df=10$ ($t=6$) and $df=22$ ($t=12$), for differences of 40%, 30%, 20%, 10%, and 0% between the respective treatment mean and the control mean. The values in the cells represent the **discriminative power** in percentage based on 200 simulated experiments.

df	$T_1 - T_6$ (40%)			$T_2 - T_6$ (30%)			$T_3 - T_6$ (20%)		
	6	10	22	6	10	22	6	10	22
LSD	52.5	57.5	67	32	41	42.5	15.5	22.5	16.5
LSDP	35	39.5	44	18	30	30.5	9	18.5	12.5
D	49.5	53	57	28.5	36	31.5	14.5	20.5	11.5
DP	35	38	41	17	28.5	23	9	16.5	9.5
SNK	30.5	27.5	19.5	12	14.5	3.5	8	8	1.5
Tu	24	25	16.5	8.5	12	3	6.5	4	1.5
B	18.5	20.5	12.5	5.5	8.5	1.5	3.5	2.5	1
Du	33	36.5	32.5	14	17.5	9.5	8	8.5	6
BM	32	35	39.5	14	19.5	17.5	7.5	9	9

df	$T_4 - T_6$ (10%)			$T_5 - T_6$ (0%)		
	6	10	22	6	10	22
LSD	9	10.5	11.5	3.75	6	5.5
LSDP	9	9	8.5	3.75	4	5
D	9	9	6	3.25	5.5	3
DP	9	7.5	5.5	3.25	4	2.5
SNK	4	3	0.5	2	2.5	0
Tu	0.5	1.5	0.5	1	0.5	0
B	0	0.5	0	0.75	0	0
Du	2.5	4	1	1.5	1.5	1
BM	6	3.5	4	2	3	3

Table 6. Results obtained when $r=6$, $CV = 20\%$, for $k=2$ and $k=4$, for groups $G_1 + G_2$, G_3 and G_4 , and for $df=12$ ($t=4$), $df=20$ ($t=6$) and $df=44$ ($t=12$), for differences of 40%, 30%, 20%, 10%, and 0% between the respective treatment mean and the control mean. The values in the cells represent the **discriminative power** in percentage based on 100 experiments from the grouping of two single experiments.

df	$T_1 - T_6$ (40%)			$T_2 - T_6$ (30%)			$T_3 - T_6$ (20%)		
	12	20	44	12	20	44	12	20	44
LSD	90	91	93	71	68	72	31	40	32
LSDP	83	80	92	56	62	72	31	38	32
D	90	87	88	68	63	60	28	35	24
DP	83	78	87	55	60	60	28	34	24
SNK	77	65	60	42	43	25	22	20	7
Tu	71	62	50	34	34	17	17	15	5
B	65	57	43	30	30	14	15	14	3
Du	78	74	75	52	46	36	20	19	12
BM	78	72	79	48	48	52	26	27	20

df	$T_4 - T_6$ (10%)			$T_5 - T_6$ (0%)		
	12	20	44	12	20	44
LSD	13	15	17	2	8	4
LSDP	13	15	17	2	8	4
D	12	14	8	2	8	4
DP	12	14	8	2	8	4
SNK	6	11	1	2	6	0
Tu	2	9	0	1	2	0
B	2	6	0	1	2	0
Du	3	11	3	1.5	2	0
BM	8	12	6	2.5	3	3

The Tables show that the **discriminative power** of Fisher's LSD and Duncan's test under the form unprotected and protected are always higher than that of other tests (because they are of "comparisonwise type").

With the exception of a few cases (that we attribute to sampling variation), the Modified Bonferroni's test exhibits **discriminative power** greater than Dunnett's and SNK's tests, and much higher **discriminative power** than Tukey's and Bonferroni's tests.

According with the Statistical Theory we observe that for larger differences (40% and 30%) all tests exhibit high **discriminative power** for all conditions.

Higher coefficients of variation (CV) are associated with lower **discriminative power** for all statistical tests.

Greater number of replications ($r=6$) exhibits higher **discriminative power** in relation to $r=3$, for all tests.

Smaller coefficient of variation (10%) and higher number of replications ($r=6$) guarantee **discriminative power** superior to 80% for almost all types of test for differences equal or higher than 20%, with significant results in 4 of 5 experiments.

When differences are smaller (20% and 10%) the **discriminative power** is lower for all tests in almost all cases.

Many research workers prefer to use the **comparisonwise type** of error when testing differences between means (CHEW, 1977).

In cases in which the error of rejecting H_0 when it is true is judged important and must be avoided the **experimentwise error** should be preferred.

For the present situation, based in results when $k=2$ and $k=4$ and for different number of treatments, replications and degrees of freedom of the residual, we may, assume that the Modified Bonferroni's test behaved better than the group of **experimentwise tests** because the new test improved the **discriminative power**,

showing higher efficiency in the detection of the significance of the differences studied.

RESUMO

O poder discriminativo de um novo teste estatístico, designado por **Teste de Bonferroni Modificado**, foi comparado com o dos testes mais utilizados na comparação de médias. Mostra-se que para k contrastes, escolhidos "a priori", entre a média de um tratamento e a do controle (testemunha) o novo teste é mais discriminativo que os testes de Student - Newman-Keuls (SNK), de Tukey (Tu), de Bonferroni (B) e de Dunnett (Du), todos eles do tipo de erro **por experimento**, sendo, porém, menos discriminativo que os testes de Fisher (LSD), o de comparação múltipla de Duncan (D) e os testes LSD e de Duncan protegidos (LSDP e DP), que são do tipo de erro **por comparação**.

Os resultados do poder discriminativo dos testes estatísticos comparados para diferentes valores de k , de coeficiente de variação (CV) e de números de repetições (r) constam das Tabelas 1 a 6.

SUMMARY

The discriminative power of a new statistical test, called **Modified Bonferroni's Test**, was compared with that of several of the most utilized statistical tests adequate for pairwise comparisons.

It is shown that for k comparisons chosen "a priori", the test is more discriminative than Student-Newman-Keuls, Tukey's, Bonferroni's and Dunnett's, all of **experimentwise** type of error, but less discriminative than Fisher's LSD test and Duncan's multiple range test, without or with protection, these of the **comparisonwise** type.

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